Types of Mathematical Models

I. Models based on relations between variables

Ch. 2:

- **Functions** y=f(x) - qualitatively: increasing/decreasing functions
  - Concept of equilibrium as intersection of graphs of functions
  - Main model: supply and demand

- **Proportion and Scale** $y \sim x$, $y \sim x^2$
  - used mainly for relating geometrical quantities, e.g. lengths, areas, volumes

- **Dimensional Analysis** $y = k x_1^{\alpha} x_2^{\beta} \ldots$
  - Relations between physical variables; units must match

    Example: Einstein's law $E=mc^2$
  - widely used in fluid mechanics

Chs. 3,4: **Optimization Techniques**

- Find minimum (or maximum) of $f(x_1,x_2,\ldots)$ subjected to equality and/or inequality constraints,

  e.g. $x^2+y^2=1$, $x\geq 0$, $y\leq 0$, $3x+2y\leq 5$

- Mainly business problems, e.g. how many bottles of white wine and red wine a winemaker should produce to maximize the sales revenue.
- Ch. 3: Linear Optimization Problems,
- Ch.4: Nonlinear Optimization Problems

Ch. 5: **Data Fitting** Model: $y=f(x,w)$; w=set of parameters

- Find optimal w for given data set
- Main methods: Least squares and spline interpolation

II. Dynamical Systems Models

Variables vary in time

Ch. 6: **Difference Equations** $x_{n+1}=f(x_n)$, $n=0,1,2,\ldots..$ (DE)

Other types of time evolution equations

Ordinary Differential Equations $\frac{dx}{dt} = f(x)$ (ODE)

Delay Differential Equations $\frac{dx(t)}{dt} = f\big(x(t)\big) + g(x(t-\tau))$

Partial Differential Equations, Example: $\frac{\partial T}{\partial t} = D\,\frac{\partial^2 T}{\partial x^2}$

Relation between (ODE) *and* (DE):

Set $x_n=x(n\tau)$, τ: fixed small time step. Then

$$x_{n+1}=x((n+1)\tau)=x(n\tau+\tau) \approx x(n\tau)+ \tau dx(n\tau)/dt=x(n\tau)+ \tau f(x(n\tau))= x_n+ \tau f(x_n)$$

III. Simulation Modeling

Analytically Intractable Models

Ch. 7 Not discussed in class

Examples

- Weather Prediction (atmospheric science models)
- Best Strategy for playing black Jack (see Ch. 7)
- Complex ecological models (e.g. for African Savannas)
- Cellular automata models for fluid flow