

## 2.3: Models of Motion: Gravity Force and Air Resistance

- **Gravity Force:**  $F_g = -mg$ 
  - $g = 32 \text{ ft/s}^2 = 9.8 \text{ m/s}^2$
- **Air Resistance:**  $F_{\text{air}} = R(v)$ 
  - **Linear Model:**

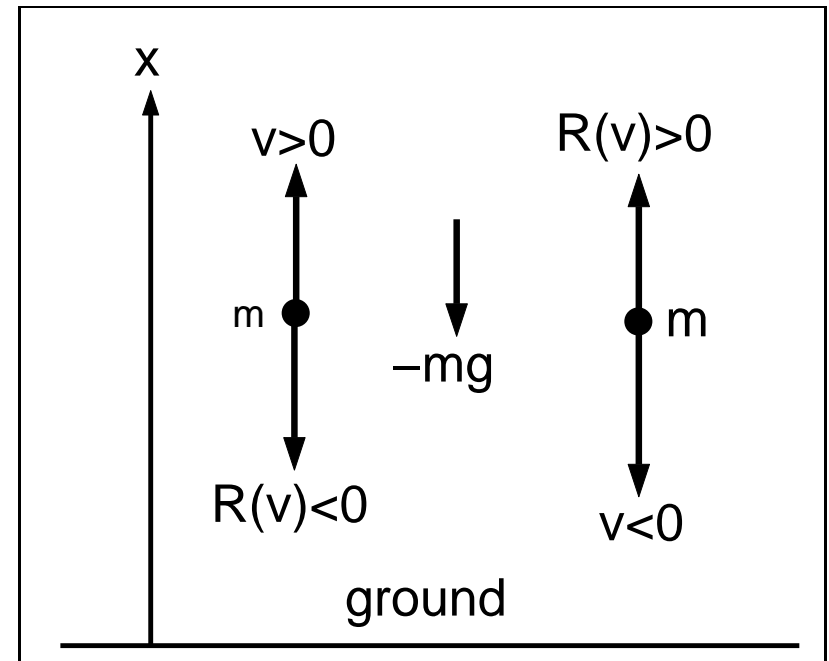
$$R(v) = -kv$$

- $[k] = \text{mass/time}$
- valid for small velocities
- **Quadratic Model:**

$$R(v) = -k|v|v$$

- $[k] = \text{mass/length}$
- valid for larger velocities

- We treat only the linear model.



## Solution without Air Resistance

$$v' = -g, \quad x' = v$$

$$\Rightarrow v(t) = -gt + v_0$$
$$x(t) = -gt^2/2 + v_0t + x_0$$

$$\Rightarrow t = (v_0 - v)/g$$
$$x = (v_0^2 - v^2)/(2g) + x_0$$

$$\Rightarrow v^2 = v_0^2 + 2g(x_0 - x)$$

Max Height (if  $v_0 > 0$ ):

$$v = 0 \Rightarrow t_{max} = v_0/g$$
$$x_{max} = x_0 + v_0^2/(2g)$$

Ground Hit:  $x = 0 \Rightarrow$

$$v_g = -\sqrt{v_0^2 + 2gx_0} \text{ (impact velocity)}$$
$$t_g = (v_0 + \sqrt{v_0^2 + 2gx_0})/g$$

### Example 1:

Ascending balloon, velocity  $15 \text{ m/s}$ .  
At height  $100 \text{ m}$  package is dropped.  
When does package reach ground?

$$g = 9.8 \text{ m/s}^2$$

Initial Values:

$$x_0 = 100 \text{ m}, \quad v_0 = 15 \text{ m/s}$$

$$\Rightarrow t_{max} = 1.5 \text{ s}$$
$$x_{max} = 111.5 \text{ m}$$
$$v_g = 46.7 \text{ m/s}$$
$$t_g = 6.3 \text{ s}$$

## Solution with Air Resistance

$$v' = -g - (k/m)v$$

In CN 2.2, Example p.5, we showed:

$$\begin{aligned} y' &= ry + a, \quad y(0) = y_0 \\ \Rightarrow y(t) &= (y_0 + a/r)e^{rt} - a/r \end{aligned}$$

Here:  $y = v$ ,  $a = -g$ ,  $r = -k/m$

$$\Rightarrow v(t) = (v_0 + gm/k)e^{-kt/m} - gm/k$$

Integrate this to find  $x$ :

$$\begin{aligned} x(t) &= \int_0^t v(t')dt' + x_0 \\ &= \frac{m}{k}(v_0 + gm/k)(1 - e^{-kt/m}) \\ &\quad - (gm/k)t + x_0 \end{aligned}$$

Terminal Velocity:

$$v_{term} \equiv \lim_{t \rightarrow \infty} v(t) = -gm/k$$

**Example 2:** (see text, Example 3.8)  
 $m = 2 \text{ kg}$ ,  $k = 4 \text{ kg/m}$  ( $g = 9.8 \text{ m/s}^2$ )  
 Initial values:  $x_0 = 250 \text{ m}$ ,  $v_0 = 0$

*Question:*

Time of ground hit? Impact velocity?

*Answer:*

Ground hit  $\rightarrow$  equation for  $t = t_g$ :

$$\begin{aligned} 0 &= g(m/k)^2(1 - e^{-kt/m}) - (gm/k)t + x_0 \\ &= 2.45(1 - e^{-2t}) - 4.9t + 250 \\ &= 252.45 - 2.45e^{-2t} - 4.9t \end{aligned}$$

Equation solver  $\rightarrow t_g \approx 51.52 \text{ s}$

Impact velocity:

$$v_g = v(t_g) = 4.9(e^{-2t_g} - 1) \approx -4,9 \text{ m/s}$$

*Without air resistance:*

$$\begin{aligned} t_g &= \sqrt{2x_0/g} \approx 7.14 \text{ s} \\ v_g &= -\sqrt{2gx_0} \approx -44.3 \text{ m/s} \end{aligned}$$

Graphs for Example 2:

